

I know what you're thinking, "I thought there was only one line between any two points. How could there be more than one distance?" In the first place, this statement is only true for Euclidean geometry. In the second, the surface which contains the loxodrome we have described consists of 2 dimensions which are not included in the usual set of 3 dimensions of real space. After all, these are the dimensions into which length disappears when relativity says it "contracts".

Even without these exotic descriptions of higher dimensional space, consider the following property of a loxodrome in ordinary 3-dimensional space. The arc length of the spiral is directly proportional to the radius. Given any two points, we can connect them with a loxodrome of any tilt angle. Since the arc length of a tilted spiral is also directly proportional to the secant of the tilt angle, we have already established that an infinite number of spirals span any pair of points. But such a path would clearly not be a straight line.

If, instead of a single spiral, we span the distance with 2 spirals, end to end. The arc length of each one of these smaller spirals is exactly $\frac{1}{2}$ the arc length of the original spiral. Since there are 2 spirals, the total arc length is unchanged. From this case, we deduce that we can repeat this process for every segment formed by dividing a larger one in half. Regardless of how many beads are strung together, the total arc length remains the same. In theory, this is true, even as n approaches infinity. The effective diameter of each contributing bead is still larger than the diameter by the scale factor, $\sec(\text{tilt})$, and the total effective diameter is still larger by the same factor.

While measurements parallel to the original segment do not change in length, the actual radius of each bead is reduced by the number of beads. With 2 beads, the radius is $\frac{1}{2}$ the original. As n approaches infinity, this radius approaches zero. In the limit, the string of beads which connects 2 points is indistinguishable from the line which does the same, but because of the tilt angle, it is longer by the factor, $\sec(\text{tilt})$. If we compare a string of beads with an effective length of 1 unit to a straight line of 1 unit, the string of beads will be contracted by the factor, $\cos(\text{tilt})$. When $v = c \sin(\text{tilt})$, this is exactly what is predicted by Special Relativity.